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HEAT AND MASS TRANSFER BETWEEN MOIST SOLIDS AND AIR

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Abstract—Heat transfer at the evaporation of a liquid from porous capillary solids differs from heat transfer at the evaporation of a liquid from a free surface. The Nusselt number depends in the process of drying of moist solids not only on the Reynolds number, but also on parametric criteria which characterize the influence of mass transfer on heat transfer. The empirical relations establishing the similarity of criteria for heat and mass transfer are determined on the basis of experimental data.

Résumé—La transmission de chaleur au cours de l'évaporation d'un liquide à partir d'un corps à porosité capillaire diffère de celle qui se produit au cours de l'évaporation d'un liquide à partir d'une surface libre. Le nombre de Nusselt dans le séchage des corps humides ne dépend pas uniquement du nombre de Reynolds mais aussi d'un critère paramétrique qui caractérise l'influence du transport de masse sur le transfert de chaleur. Les relations empiriques, qui établissent la similitude du critère pour les transports de chaleur et de masse, sont déterminées à partir des données expérimentales.

Zusammenfassung—Der Wärmeübergang bei der Verdampfung einer Flüssigkeit an kapillar-porösen Körpern unterscheidet sich von dem an einer freien Oberfläche. Beim Trocknen feuchter Stoffe hängt die Nusseltzahl nicht nur von der Reynoldszahl ab, sondern von Kenngrössen, die den Einfluss der Stoffübertragung auf die Wärmeübertragung charakterisieren. Auf Grund von Versuchswerten werden die entsprechenden Beziehungen aufgestellt.

Аннотация—Теплообмен в процессе испарения жидкости из капиллярно-пористых тел отличается от теплообмена при испарении жидкости со свободной поверхности. Тепловой критерий Нуссельта в процессе сушки влажных тел зависит не только от критерия Рейнольдса, но и от ряда параметрических критериев, характеризующих влияние массообмена на теплообмен.

На основе экспериментальных данных установлены эмпирические соотношения между критериями подобия и тепломассопереноса.

DURING the process of drying, heat and mass transfer from the surface of a solid into the surrounding moist air is complicated. From a number of previously published works one can see that the drying of moist solids is analogous to the evaporation of liquids from a free surface. However, the identity of heat and mass transfer processes in these two cases did not appear to be corroborated by later works of greater accuracy. The well-known Lewis relation between the heat transfer coefficient α and the coefficient of mass transfer

$$\frac{\alpha}{\beta} = c\gamma \qquad (1)$$

received no confirmation [1]. Here c is the heat capacity of moist air and γ its density.

All attempts to insert some corrections into relation (1) ended in failure, since the rated formulae did not give correct result.

For non-isothermal conditions the total flow of mass is equal to the sum of diffusion, thermodiffusion and molar flows [2].

For the usual conditions evaporation the concentration diffusion is of greatest significance (from 90 to 84 per cent); the magnitude of the molar flow reaches 16 per cent and the portion of the thermodiffusion flow makes up only 1 per cent. In most cases except those when mixtures contain hydrogen it is possible to neglect thermodiffusion.

A model of the molecular mechanism of heat transfer, not complicated by mass transfer, may be presented as follows: near the surface of heat transfer the laminar layer (laminar sublayer) is observed where the transfer of heat is only caused by molecular thermal conductivity; then there is a transient zone, where heat transfer goes on both by heat conduction and by convection (the laminar layer with slightly expressed turbulence); at last follows the turbulent kernel of the flow, where heat transfer goes on by means of hydrodynamic mixing (molar heat transfer).

In case of heat transfer accompanied by mass transfer the mechanism of the whole process alters somewhat. The presence of the mass flow in a direction normal to the surface from which evaporation proceeds accelerates molecular heat transfer, thus causing an increase in the heat transfer coefficient, other conditions remaining equal.

Besides, at high intensities the process of phase conversion, accompanied by an increase in volume of the substance, considerably influences an increase of the coefficient of heat conduction as well.

The influence of mass transfer on heat transfer may be expressed most precisely by the criterion dependence obtained by the transformation of the system of differential equations describing the law of heat transfer on the surface of a solid and the temperature field of air flowing along the surface.

We shall now consider the mechanism of mass transfer from the surface of a liquid into the surrounding medium (moist air). The moist air may be regarded as a binary vapour–gas mixture, consisting of vapour and dry air.

The vapour diffuses from the surface of a liquid under the influence of the relative concentration gradient $\partial\gamma_{10}/\partial y$, where y is the direction of a normal to the surface of the liquid, γ_{10} is the relative vapour concentration: $\gamma_{10} = \gamma_1/\gamma$, where γ_1 is vapour concentration (vapour mass per unit volume of mixture). The air diffuses in the reverse direction. In a steady state [3] the diffusion flows of vapour and of air are equal, i.e.

$$D_{12}\gamma\frac{\partial\gamma_{10}}{\partial y} = -D_{21}\gamma\frac{\partial\gamma_{20}}{\partial y}\,; \tag{2}$$

for the coefficients of the interdependent diffusion of vapour into air and of air into vapour are equal ($D_{12} = D_{21}$), and the gradients of relative concentration of air, $\partial\gamma_{20}/\partial y$, and of vapour. $\partial\gamma_{10}/\partial y$, are equal in magnitude, but opposite in direction ($\gamma_{10} + \gamma_{20} = 1$). Molar motion of moist air (Stefan's flow), with a linear velocity w_c, arises owing to the presence of an obstacle for the diffusion of air, namely, the surface of the liquid at a constant barometric pressure. This molar motion is a convective flow, and increases the vapour transfer by $\gamma_1 w_c$ (the convective component of vapour transfer).

The total specific vapour flow j_1 (kg/m² hr) is equal to

$$j_1 = \gamma\gamma_{10}w_c - D_{12}\gamma\frac{\partial\gamma_{10}}{\partial y} \tag{3}$$

The resultative air flow is zero ($j_2 = 0$)

$$0 = \gamma\gamma_{20}w_c - D_{21}\gamma\frac{\partial\gamma_{20}}{\partial y} \tag{4}$$

From the equation (4) w_c can be determined; using formula (2), we obtain

$$w_c = -\frac{D_{12}}{\gamma_{20}}\frac{\partial\gamma_{10}}{\partial y} \tag{5}$$

On the basis of this equation, formula (3) can be written as:

$$j_1 = \gamma\gamma_{10}w_c + \gamma\gamma_{20}w_c = \gamma w_c\,; \tag{6}$$

whence for the linear velocity of convective vapour transfer we obtain the following expression

$$w_c = \frac{j_1}{\gamma} \tag{7}$$

Usually the hydrodynamics of heat transfer for the forced motion of air are characterized by a Reynolds number, equal to the ratio of the product of the linear velocity of air along the surface w by the characteristic size l to the coefficient of the kinematic viscosity ν:

$$Re = \frac{wl}{\nu}$$

In the case of heat transfer complicated by mass transfer it is necessary to insert the additional criterion K, which characterizes the convective vapour transfer from the surface of a liquid into the surrounding air:

$$K = \frac{w_c l}{\nu} = \frac{j_1 l}{\eta} \tag{8}$$

where η is the coefficient of dynamic viscosity ($\eta = \nu\gamma$). Then the relation between the Nusselt number Nu and criteria Re and K may be written for moist air as:

$$Nu = ARe^n (1 + K)^m \tag{9}$$

which leads to the expression $Nu = ARe^n$ at $K = 0$ (the absence of evaporation); i.e. the usual expression of Nu for dry material.

The presence of a unit in the factor $(1 + K)$ gives universality to the formula (9), as at $K = 0$ the factor $(1 + K)$ does not become zero. At the same time K usually has such an order of magnitude (up to 13,500) that a unit is of a negligibly small value. The advantage of the present formula lies in the fact that it holds true for the total duration of all drying processes. From this point of view it may be widely used for thermophysical investigations of the drying process and for an analysis of the operation of working dryers. For designing, the application of this formula should be excluded, as here it is necessary to know beforehand the value of the drying intensity j_1, an unknown quantity.

The members of the Luikov school have suggested in their later works taking into account the influence of the mass transfer process on the heat transfer coefficient for convective drying, using the Guchman criterion, which characterizes the potential of drying. On the basis of experimental work A. V. Nesterenko suggested determining the heat transfer coefficient for the evaporation of water from the open surface according to the formula:

$$Nu_f = Nu_0 + ARe_f^n \, Pr_f^{0.33} \, Gu^{0.175} \tag{10}$$

where Nu_0 is the value of the Nusselt number at $Re = 0$, $Pr = \nu/a$ is the Prandtl number and $Gu = (t_m - t_w)/T_m$ is the Guchman criterion in which t_m and t_w are the temperatures of the medium and of the evaporated water (°C), respectively, T_m is the medium's temperature (°K). The index of f for the criteria in formula (10), shows that their values should be taken for the average temperature of gas flow. A and n are coefficients the values of which, depending on Re, are given in Table 1.

For $Nu > 80$ the value $Nu = 2$ (at $Re = 0$) can be neglected. The criterion equations include not only the criteria obtained from the fundamental equation for connexion and boundary conditions obtained by the method developed by the similarity theory, but may also include parameter criteria, the choice of which may be made according to the physical nature of the process.

Table 1. The values of A and n in formula (10)

Re number	*A*	*n*
1–200	1·05	0·50
200–25000	0·385	0·57
25000–70000	0·102	0·73

The application of the parameter criteria may be advantageous both for determining the unknown quantity and for the application of the criterion dependences obtained in engineering.

Proceeding from these considerations we propose a universal equation, suitable for any method of heat supply to the material (convexion, radiation, contact and others) covering the entire drying process (both periods) and including initial values which usually serve in calculations applied to the process of drying:

$$Nu = ARe^n N^m \theta^k \left(\frac{W}{W_{\text{cr}}}\right)^\delta \tag{11}$$

In this equation the criterion N, as well as the parameter criterion Gu in equation (10), determines the increase of the heat transfer coefficient at the expense of the turbulization of the air flow by vapours which evaporate from the material surface:

$$N = \frac{T_m}{T_{wt}}$$

where T_m is the absolute temperature of a medium (°K) and T_{wt} is the temperature of the wet thermometer (°K).

The parameter criterion θ in the equation determines the increase of the heat transfer coefficient at the expense of the decrease of the boundary layer thickness with the rise of the irradiator temperature for radiative drying.

The parameter criterion W/W_{cr} is the ratio of the material's moisture content during the period the drop in the rate of drying is observed to the critical moisture content of the material; it takes into account the decrease of the heat transfer coefficient with the decrease of the material's moisture content during the drop in the rate of drying.

However, it should be noted that the more correct way is to insert u/u_{cr} instead of W/W_{cr} where u is the moisture content at the surface of the sample in the moment considered (when there is a drop in the rate), u_{cr} is the critical moisture content at the surface of the sample.

What we have stated above follows from these considerations: the ratio of the average moisture content of a sample in the period of time considered characterizes least of all the process of heat and mass transfer going on at the material's surface.

However, for a practical application of formula (11) the criterion W/W_{cr} has a considerable advantage as it is easier and quicker to determine the average moisture content of the material in the process of drying rather than the moisture content at its surface. Moreover for small gradients of moisture content in a sample, i.e. for combined drying by high-frequency currents and by radiation, the difference between W/W_{cr} and u/u_{cr} may be insignificant.

Calculating from experimental data for the whole process of the combined drying of materials the following formulae were derived:

for wood

$$Nu = 0{\cdot}5Re^{0\cdot5}\left(\frac{T_m}{T_{wt}}\right)^2\left(\frac{T_r}{T_m}\right)^{0\cdot4}\left(\frac{W}{W_{\mathrm{cr}}}\right)^{0\cdot3} \tag{12}$$

for clay

$$Nu = 0{\cdot}45Re^{0\cdot5}\left(\frac{T_m}{T_{wt}}\right)^2\left(\frac{T.}{T_m}\right)^{0\cdot4}\left(\frac{W}{W_{\mathrm{cr}}}\right)^{0\cdot9} \tag{13}$$

for sand

$$Nu = 0{\cdot}65Re^{0\cdot5}\left(\frac{T_m}{T_{wt}}\right)^2\left(\frac{T_r}{T_{wt}}\right)^{0\cdot4}\left(\frac{W}{W_{\mathrm{cr}}}\right)^{1\cdot8} \tag{14}$$

The various values for the coefficients A and for the exponents σ of the parameter criterion W/W_{cr} take into account the different ways in which the moisture is associated with the material during the period registering a drop in the rate of drying as well as the difference between the geometric and the real surface of evaporation in some materials as, for example, for sand.

Following this method as indicated in the formulae given below which were obtained by A. P. Bazhenov:

for convective drying of staple fabric:

$$Nu = 0{\cdot}94Re^{0\cdot5}\left(\frac{T_m}{T_{wt}}\right)^2\left(\frac{W}{W_{\mathrm{cr}}}\right)^{0\cdot45} \tag{15}$$

for radiative–convective drying of staple fabric during the period the rate of drying is constant:

$$Nu = 0{\cdot}9Re^{0\cdot5}\left(\frac{T_m}{T_{wt}}\right)^2\left(\frac{T_r}{T_m}\right)^{0\cdot42} \tag{16}$$

On the basis of experimental data B. M. Tcherkinsky obtained the following formula for the drying of a fabric

$$Nu = 0{\cdot}84Re^{0\cdot5}\left(\frac{T_m}{T_{wt}}\right)^2\left(\frac{T_r}{T_m}\right)^{0\cdot4}\left(\frac{W}{W_{\mathrm{cr}}}\right)^{0\cdot45} \tag{17}$$

S. V. Shishkov obtained a formula for the drying of asbestos–rubber rings as follows:

$$Nu = 0{\cdot}68Re^{0\cdot5}\left(\frac{T_r}{T_m}\right)^{0\cdot4}\left(\frac{W}{W_{\mathrm{cr}}}\right)^{0\cdot34} \tag{18}$$

G. A. Grushke obtained a formula for the radiative drying of ceramic plates

$$Nu = 0{\cdot}4Re^{0\cdot5}\left(\frac{T_m}{T_{wt}}\right)^2\left(\frac{T_r}{T_m}\right)^{3\cdot5}\left(\frac{W}{W_{\mathrm{cr}}}\right)^{2\cdot9} \tag{19}$$

Analogous formulae were obtained in the B. I. Pyatachkova experiments on the drying of lump peat, etc., from which one can obtain the formulae calculated for the design of dryers.

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